The Degree of Global-State Awareness in Self-Organizing Systems

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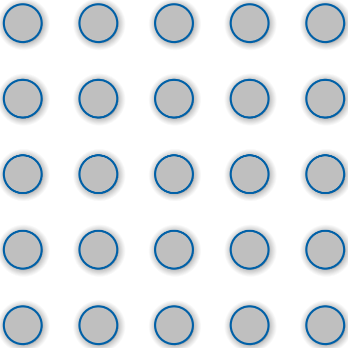
Motivation: Intrusion Detection in Sensor Networks



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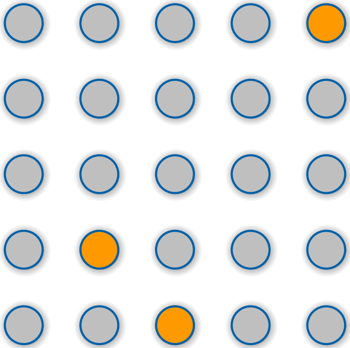

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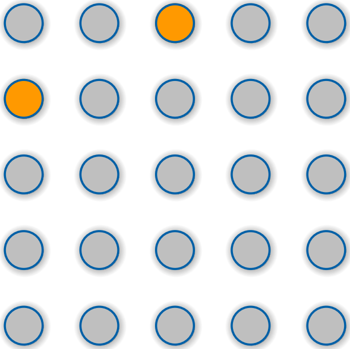

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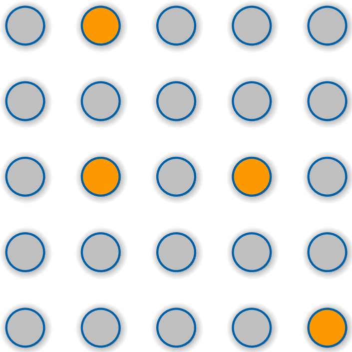

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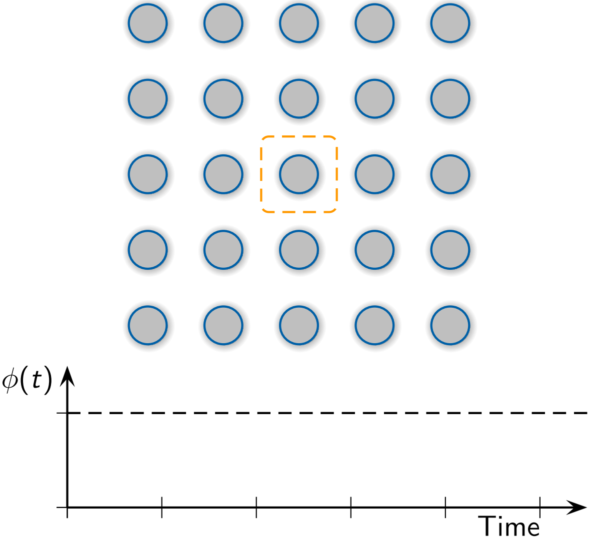

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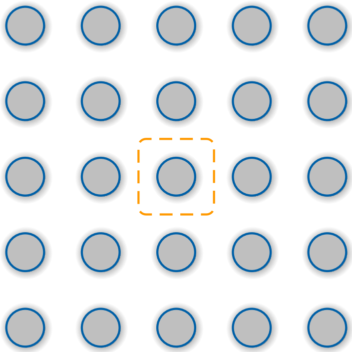

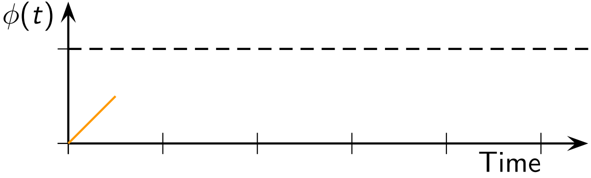

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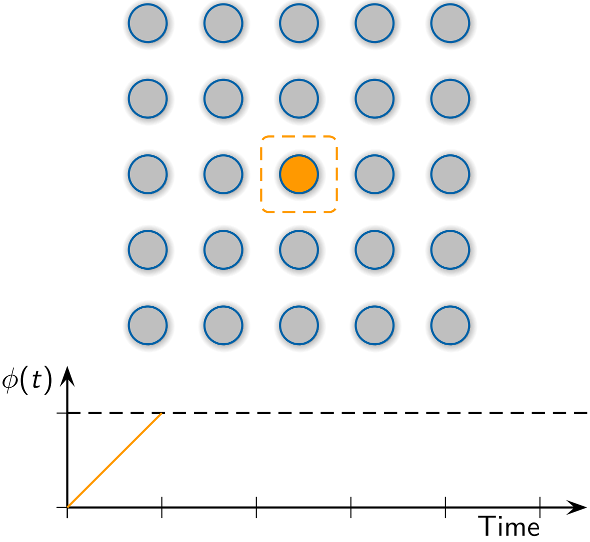

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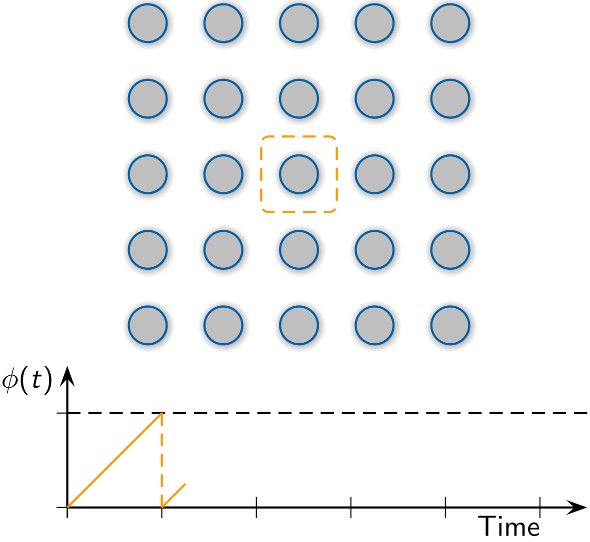

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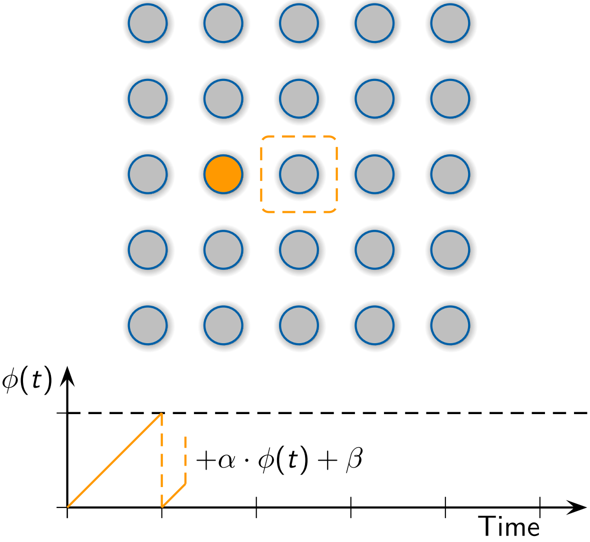

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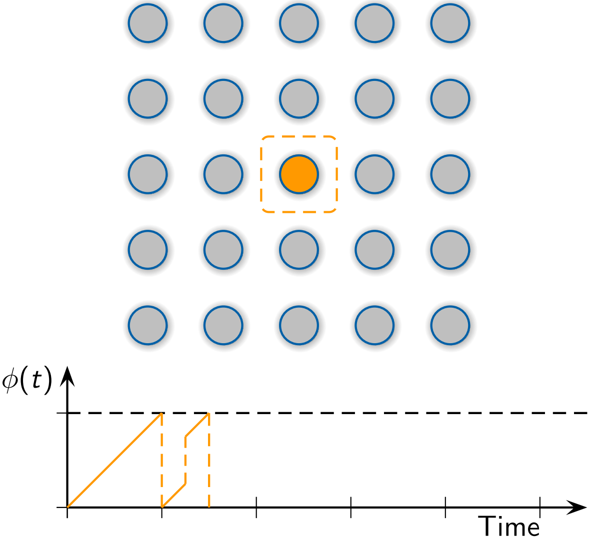

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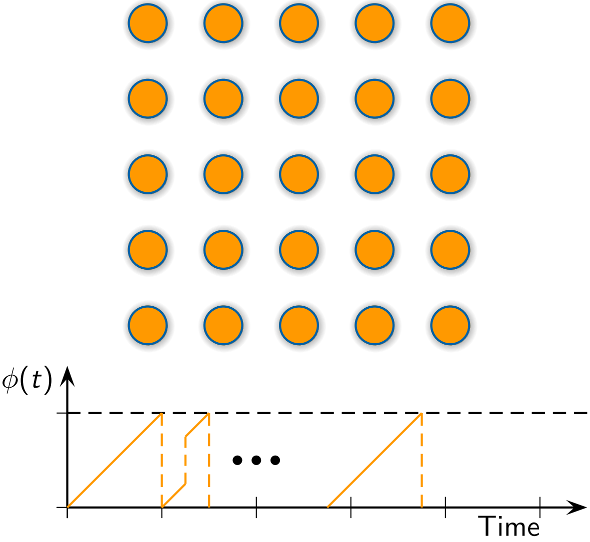

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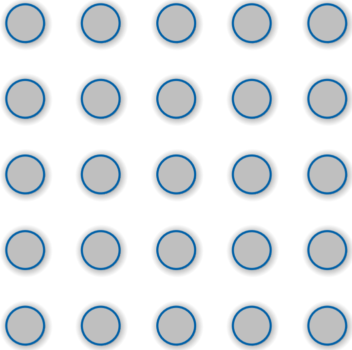

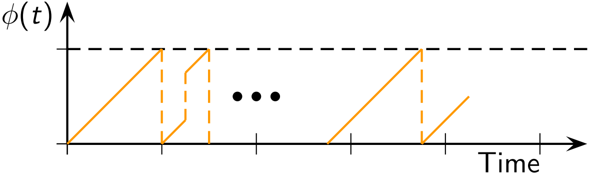

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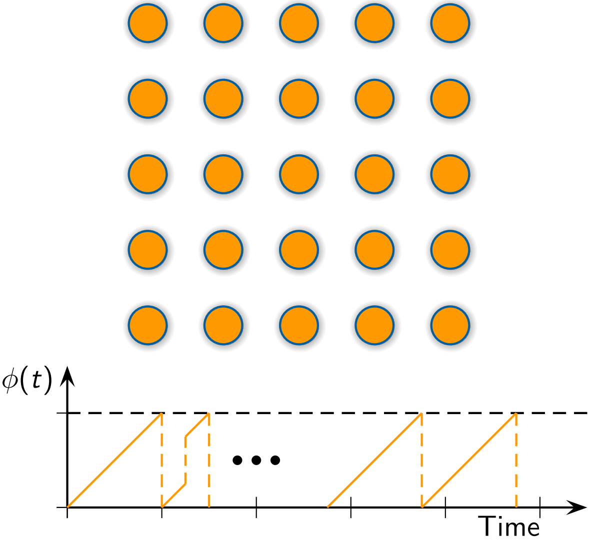
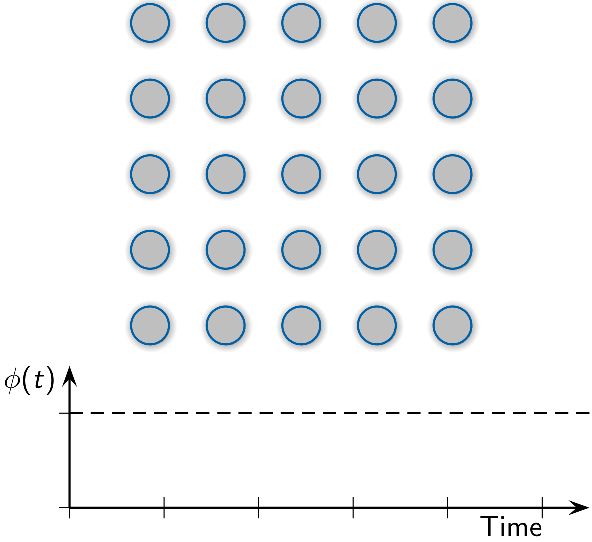
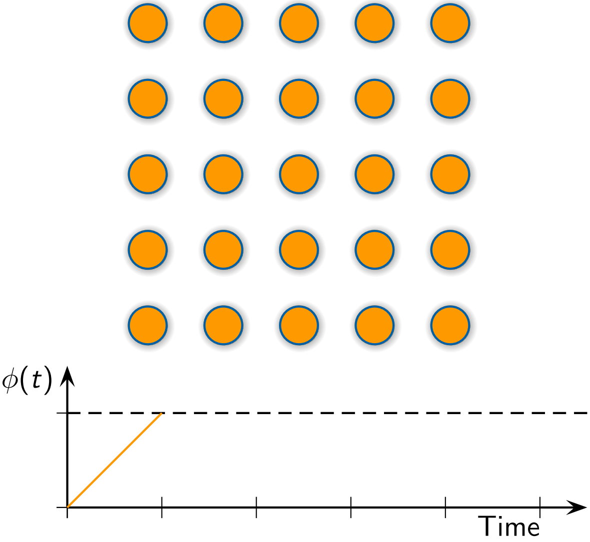




$\phi(t)$
Time



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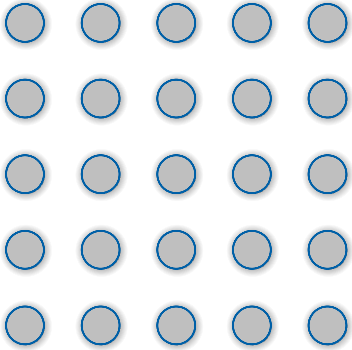

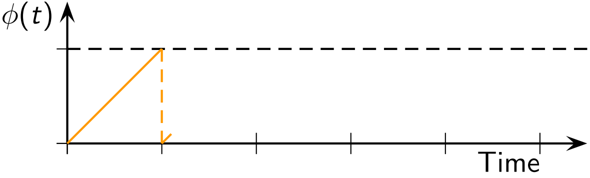

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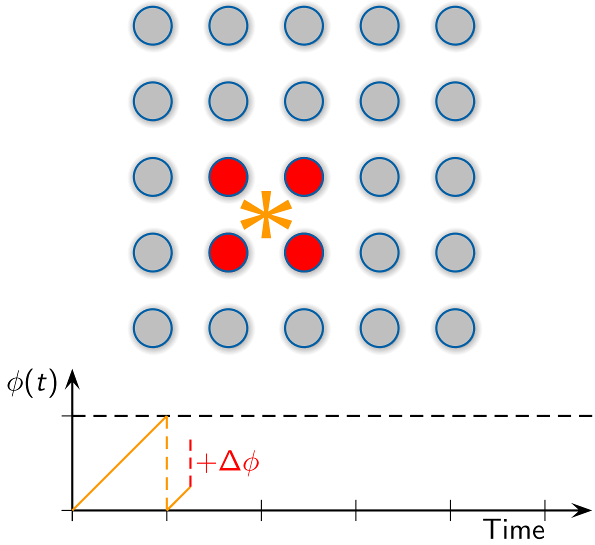

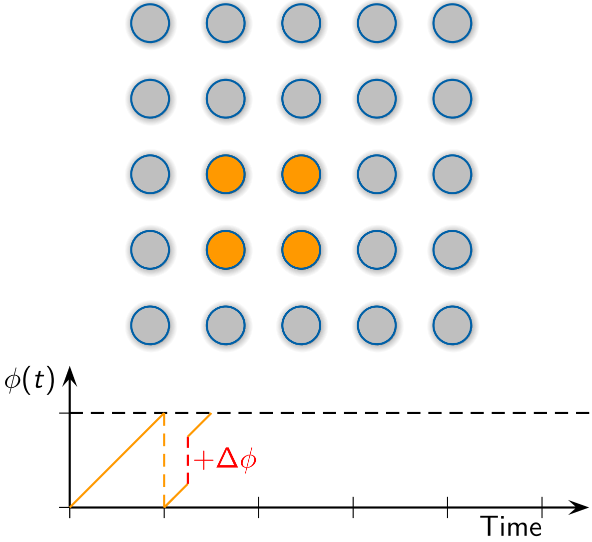
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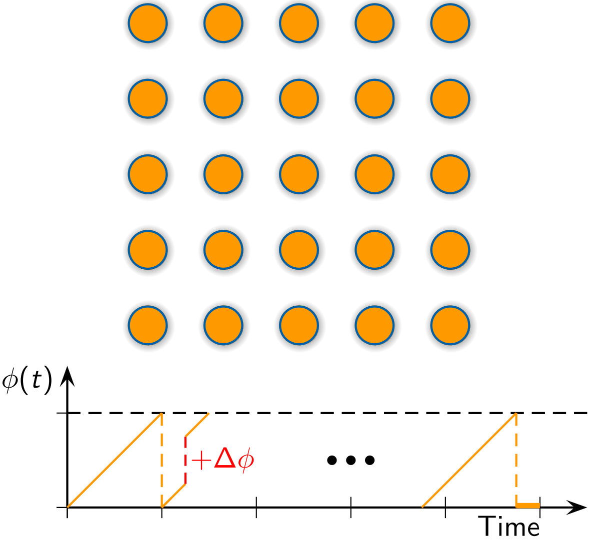

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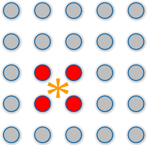
- $> 1/5$ of sensor give alarm $\Leftrightarrow$ global consensus: Intrusion! (no false-positive)
- Each sensor
  - only observes its immediate neighborhood
  - takes appropriate actions depending on global consensus
- IWSOS 08: Derive appropriate local interaction strategies that use locally accessible information
- However: Is the necessary information distributed?
- How to choose α and $\Delta\phi$?
- Degree of Global-State Awareness



- Events occur at discrete time steps $t \in \mathbb{N}_0$
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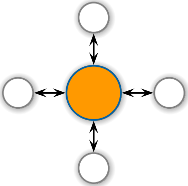

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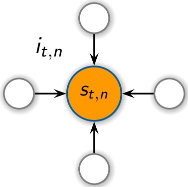

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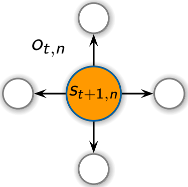

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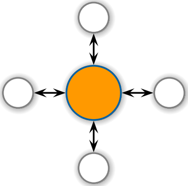

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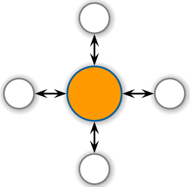
- State space of the system: $\mathbf{\Gamma}$
- Random initial configuration Γ_0 with range $\mathbf{\Gamma}_0 \subseteq \mathbf{\Gamma}$
- Random local history: $\overleftarrow{\Gamma}_{t,n}$



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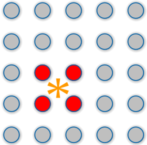

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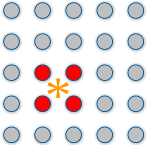

  - Partition of $\mathbf{\Gamma}_0$ into equivalence classes $\mathbf{L}$:

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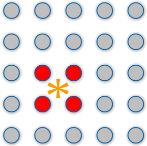
- Classification problem $\mathbf{L}$: To which extent is it possible for the system entities' to derive L?



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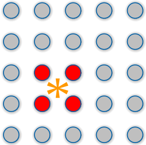

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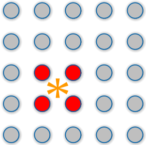

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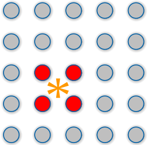

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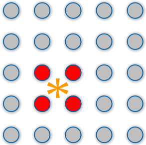

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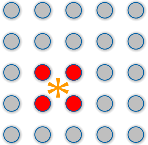

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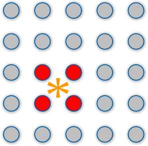

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- For the whole system:

$$\omega(\mathbf{L}) = \frac{1}{\#\mathbf{N}} \lim_{t \to \infty} \sum_{n \in \mathbf{N}} \omega_{t,n}(\mathbf{L})$$

- Initial configurations $\mathbf{\Gamma}_0$: $\phi(0) = 0$ for all sensors, except for the group of sensors with $\phi(0) = \Delta\phi$ that detected an event
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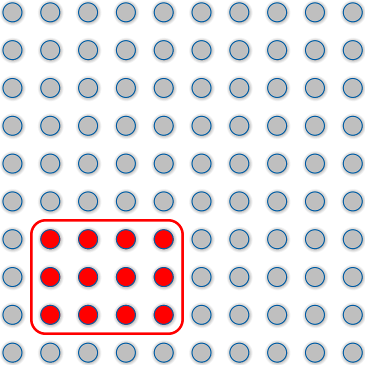

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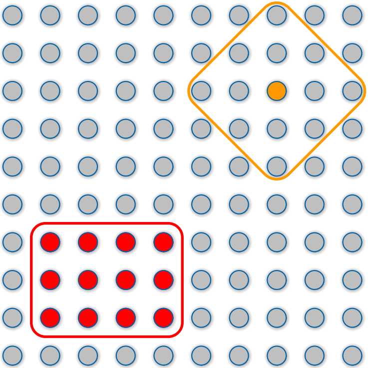
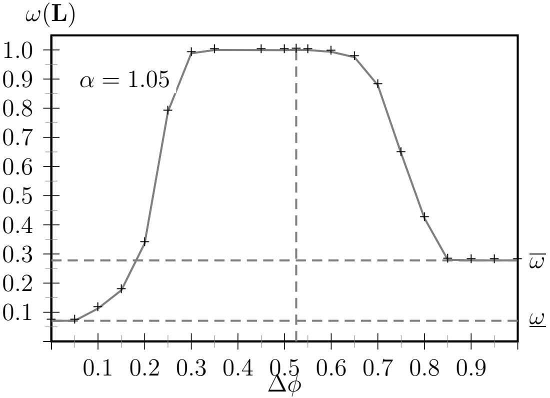
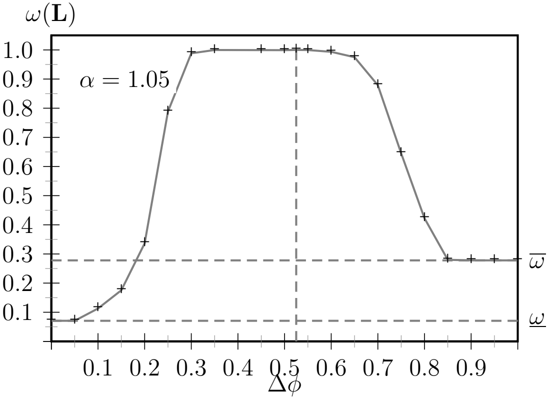
- Sensing range: Manhattan distance ≤ 2





- $\Delta\phi = 0.525$ and $\alpha = 1.05 \implies L = f(\overleftarrow{\Gamma}_{t,n})$



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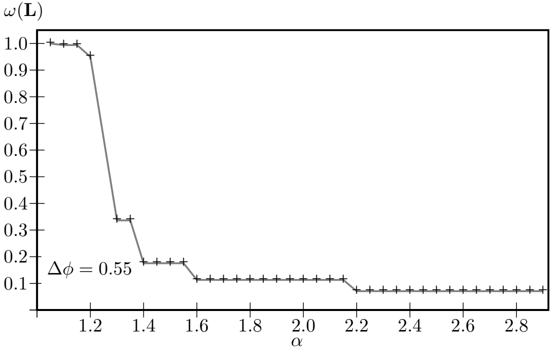
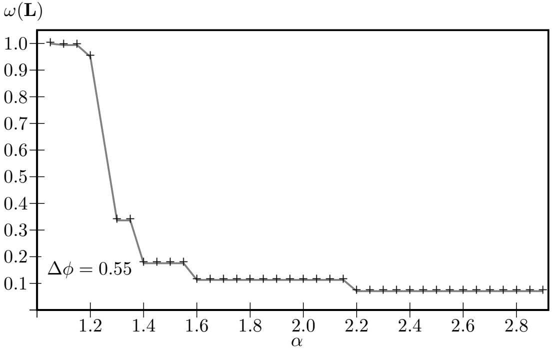
- $\overline{\omega}$, $\underline{\omega}$: detecting sensors can observe their own behavior



- Convergence time is proportional to $1/\alpha$



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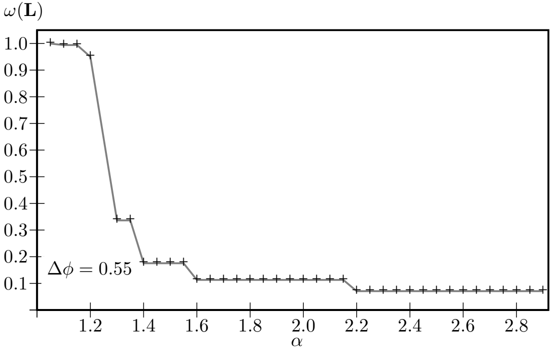
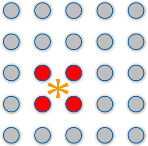
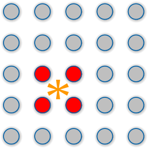


- Convergence time is proportional to $1/\alpha$
  - α too large: Information about L has no time to spread
  - α too low: Convergence time too long

- Conclusion
  - Entities observe their immediate neighborhood
  - To which extent can the entities derive global-state properties?
  - Degree of Global-State Awareness
  - Intrusion detection in sensor networks
    - Preferable system parameters
    - Influence of convergence time





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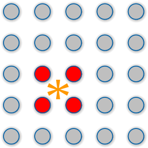

Questions?